

International Journal of Modern Physics E
 © World Scientific Publishing Company

A LAGRANGIAN FOR THE CHIRAL $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ QUARTET NUCLEON RESONANCES

V. DMITRAŠINOVIĆ

Vinča Institute of Nuclear Sciences, lab 010, P.O.Box 522, 11001 Beograd, Serbia

ATUSHI HOSAKA

Research Center for Nuclear Physics, Osaka University, Ibaraki 567-0047, Japan

KEITARO NAGATA

Department of Physics, Chung-Yuan Christian University, Chung-Li 320, Taiwan

Received (received date)

Revised (revised date)

We study the nucleon and three N^* resonances' properties in an effective linear realization chiral $SU_L(2) \times SU_R(2)$ and $U_A(1)$ symmetric Lagrangian. We place the nucleon fields into the so-called "naive" $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ and "mirror" $(0, \frac{1}{2}) \oplus (\frac{1}{2}, 0)$ (fundamental) representations of $SU_L(2) \times SU_R(2)$, two of each - distinguished by their $U_A(1)$ chiral properties, as defined by an explicit construction of the nucleon interpolating fields in terms of three quark (Dirac) fields. We construct the most general one-meson-baryon chiral interaction Lagrangian assuming various parities of these four nucleon fields. We show that the observed masses of the four lowest lying nucleon states can be well reproduced with the effective Lagrangian, after spontaneous symmetry breakdown, without explicit breaking of $U_A(1)$ symmetry. This does not mean that explicit $U_A(1)$ symmetry breaking does not occur in baryons, but rather that it does not have a unique mass prediction signature that exists e.g. in the case of spinless mesons. We also consider briefly the axial couplings with chiral representation mixing.

1. Introduction

Chiral symmetry, as one of the symmetries of QCD, is a key to understanding the dynamics of the strong interaction. In the real world chiral symmetry is spontaneously broken, and plays a dynamical role in various scattering processes involving the Nambu-Goldstone bosons. Hadrons are then classified only according to the residual vector symmetry and the full chiral symmetry can be conveniently represented by the non-linear realization. Yet, as pointed out by Weinberg¹, it still makes sense to talk about irreducible representations of the complete chiral symmetry group, and consider hadrons as mixtures of a (small) number of such chiral multiplets. If so, one can use the chiral symmetry as an algebraic symmetry that puts constraints on physical observables, such as masses and coupling constants. Furthermore, as chiral symmetry is restored at high temperature and density, the

2 Authors' Names

change in hadron properties can be viewed as a function of the change in the representation mixing. The present paper is based on this point of view, where we extend some previous work by Christos² for $N_f = 2$, to include chiral “mirror” nucleon fields, with a view to ultimately extending it to $N_f = 3$, where some pioneering work has been done in Refs.^{3,4,5}. So, this paper may be viewed, to some extent, as an intermediate pedagogical step on the way to the full-blown problem. Yet, we shall see that this simpler $N_f = 2$ case contains most of the features of the $N_f = 3$ case, and that certain characteristic problems become clearer without the algebraic complexity of the $N_f = 3$ case.

It is, therefore, important to determine the starting point here, *viz.* the chiral multiplets of bare hadrons, in particular of baryons, which are then mixed by the interactions to produce the physical/dressed hadrons. Phenomenologically, the success of the quark model implies the dominance of three valence quark component in nucleon states^a. Our strategy is therefore to choose nucleon chiral multiplets guided by the nucleon fields written in terms of three quarks. Strictly speaking, in the broken symmetry world the chiral structure of the interpolating field is not identical to that of the physical state that is coupled to the field; generally, the latter could be more complicated than the former. In this sense, our choice of chiral multiplets is perhaps the simplest one consistent with QCD.

The chiral multiplets associated with the lowest Fock space components of the $I(J) = \frac{1}{2}(\frac{1}{2})$ (nucleon) fields are $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ and $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$. For higher Fock space components (e.g. pentaquarks, septaquarks etc.) one finds chiral multiplets $(\frac{3}{2}, 1) \oplus (1, \frac{3}{2})$, $(\frac{5}{2}, 2) \oplus (2, \frac{5}{2})$ etc., all of which we shall ignore here. Indeed we shall not even include the $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ multiplet, which appears only with non-local three-quark interpolating fields. The classification of local three-quark nucleon fields into chiral multiplets has been recently worked out in Ref.⁶. Following up on this, we use the $U_A(1)$ and $SU_L(2) \times SU_R(2)$ chiral (here and in what follows we shall refer to them as the Abelian and the non-Abelian chiral symmetries, respectively) transformation properties of the two independent $J = \frac{1}{2}$ local nucleon fields to write down and classify the possible nucleon-meson interaction terms in the present paper.

Nucleon fields containing no derivatives are natural for the even-parity ground state nucleons; in that case only the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ non-Abelian multiplet is allowed by the Pauli principle⁶. It turns out, however, that there are two such linearly independent fields. Their linear combinations form different (“opposite”) irreducible representations of the $U_A(1)$ symmetry: one with the axial baryon number -1 and another with axial baryon number $+3$ ^b⁷. We shall use these properties to classify the nucleon-meson interaction terms in the present paper.

For odd-parity nucleon excited states, on the other hand, fields with (at least)

^aA reasonable reproduction of the nucleon ground state properties in the quenched lattice QCD may also be seen as a validation of this point of view.

^bJido et al⁷ also noticed the curious $U_A(1)$ transformation properties of one linear combination of the nucleon fields, but did not mention that it was an Abelian “mirror” assignment.

one derivative appear natural. Once we allow for one space-time derivative to exist, we find nucleon fields with chiral properties that are opposite, or complementary, i.e. “mirror” to the non-derivative “naive” ones^{8,9}, e.g. the $(0, \frac{1}{2}) \oplus (\frac{1}{2}, 0)$. This fact allows one to explicitly construct the interactions of the four different nucleon fields with chiral mesons that can account for both the masses and decays of the lowest-lying even- and odd-parity nucleon resonances.

As a specific example, we choose four particular nucleon fields, forming a $U_A(1)$ chiral nucleon quartet, that we identify with the four lowest-lying nucleon resonances: the nucleon-Roper even-parity pair and the $N^*(1535)$ and $N^*(1650)$ odd-parity resonances. We estimate the coupling strengths from the nucleon masses. Our method applies equally well to any, and not just the lowest-lying, $U_A(1)$ chiral nucleon quartet, i.e., pair of nucleon parity doublets.

It turns out that there are many allowed one-meson-baryon interaction terms, even in the limit of exact $SU_L(2) \times SU_R(2)$ and $U_A(1)$ chiral symmetries. This large latitude in the theory stems from the existence of cubic-meson-field interactions, first noticed by Christos², which turn into one-meson-baryon interaction terms after spontaneous breakdown of the chiral symmetry. The existence of cubic-meson-field interactions, in turn, is a consequence of the existence of nucleon fields with the Abelian axial charge +3. We shall show that this fact implies that one does not need explicit $U_A(1)$ symmetry breaking terms due to the $U_A(1)$ anomaly in the nucleon sector to describe the nucleon masses and decays: they can all be described without taking explicit $U_A(1)$ symmetry breaking into account. This is in contrast with the case of purely linear-meson-field interactions, which requires some explicit $U_A(1)$ symmetry breaking terms in the nucleon sector to describe the nucleon masses.

Consequently, the explicit $U_A(1)$ symmetry breaking, or its restoration, do not need to have an impact on the nucleon resonance spectrum. Note, however, that it is always possible to incorporate the explicit $U_A(1)$ symmetry breaking effects by including e.g. the 't Hooft interaction in the mesonic sector, or even in the baryon sector of the Lagrangian without “spoiling” the pattern/ordering of baryon masses^c. This would not be the case if there were only one kind of nucleon field, i.e., it is a consequence of the existence of two kinds of nucleon fields with different axial baryon numbers.

It must be stated that a closely related study of the nucleon ground state and one resonance has been done by Christos in Ref.², but without introducing the nucleon mirror fields. Indeed, he studied only the two independent nucleon non-derivative fields and related their masses to certain $U_A(1)$ symmetry conserving interactions. One distinct disadvantage of models without mirror nucleons is that phenomenologically they can not describe the one-pion decays of nucleon resonances at the tree level¹⁰. This motivates us to consider a class of models that includes mirror nucleons, which task we complete in the present paper. There is another kind of baryon chiral multiplet mixing: the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ and $(1, \frac{1}{2}) \oplus (1, \frac{1}{2})$ mixing.

^ctheir absolute values may change, of course, but then they can be renormalized

4 *Authors' Names*

That is possible only when one allows derivatives in the three-quark interpolating fields, and will be dealt with in a separate paper ¹¹.

This paper falls into five sections. After the Introduction, in Sect. II we give a reminder of the basic facts regarding the nucleon fields and their chiral transformation properties, as well as derive the chiral properties of the new derivative fields. Then in Sect. III we examine the nucleon-meson couplings and classify the interaction terms according to their symmetry properties for one particular pair of nucleon fields. In Sect. IV we calculate some of the basic predictions of this effective interaction. Finally, in Sect. V we summarize and draw our conclusions.

2. Three-quark nucleon fields

Firstly we examine the $SU_L(2) \times SU_R(2)$ and $U_A(1)$ transformation properties of various quark trilinear forms with quantum numbers of the nucleon. This leads us to two pairs of independent even and odd-parity nucleon resonances with particularly simple $SU_L(2) \times SU_R(2)$ and $U_A(1)$ transformation properties.

Five non-vanishing, apparently different baryon local fields have been explicitly constructed from three quark fields without derivatives, such that the nucleon quantum numbers are properly reproduced. Only two out of these five local trilinear fields, are linearly independent, however ¹². Their propriety for the job at hand can be tested in various ways: one way is by study of their chiral symmetry transformation properties, another is by directly applying them to the study of physical quantities, e.g. by QCD sum rule or lattice QCD. In the QCD sum rule approach, it was shown that one linear combination of the two independent nucleon fields couples predominantly to the ground state nucleon, while the one with orthogonal weights couples to the lowest-lying odd-parity nucleon resonance $N^*(1535)$ ^{13,7}.

2.1. Even-parity nucleon fields

For completeness' sake we show the five non-derivative objects involving three quark fields

$$N_1^+ = \epsilon_{abc}(\tilde{q}_a q_b) q_c, \quad (1)$$

$$N_2^+ = \epsilon_{abc}(\tilde{q}_a \gamma^5 q_b) \gamma^5 q_c, \quad (2)$$

$$N_3^+ = \epsilon_{abc}(\tilde{q}_a \gamma_\mu q_b) \gamma^\mu q_c, \quad (3)$$

$$N_4^+ = \epsilon_{abc}(\tilde{q}_a \gamma_\mu \gamma^5 \vec{\tau} q_b) \cdot \vec{\tau} \gamma^\mu \gamma^5 q_c, \quad (4)$$

$$N_5^+ = \epsilon_{abc}(\tilde{q}_a \sigma_{\mu\nu} \vec{\tau} q_b) \cdot \vec{\tau} \sigma^{\mu\nu} q_c, \quad (5)$$

which are assigned as even-parity (as indicated by the superscript +), spin 1/2, isospin 1/2 ("nucleon") fields. Here indices a, b and c label the color of the three quarks, whereas the Pauli matrices τ_i operate in the isospin space, and q is the light quark iso-doublet; we also define the \tilde{q} to stand for $q^T C \gamma_5 i \tau_2$ in shorthand notation:

$$\tilde{q} \equiv q^T C \gamma_5 i \tau_2,$$

where $C = i\gamma^2\gamma_0$ and τ_2 is the second Pauli matrix in isospin space.

Ioffe¹² used Fierz transformations to show that there are two independent nucleon fields. Thus, one may use various combinations of independent fields among Eqs. (1) - (5) for the computation of two point correlation functions, but the linearly independent choices are all equivalent, for instance, to Eqs. (1) - (2).

We shall try and systematize the nucleon fields according to both their Abelian and non-Abelian chiral transformation properties and show that this classification lends new meaning to certain concepts introduced into the linear Gell-Mann–Levy sigma model some time ago. As mentioned in the Introduction, Lee, DeTar and Kunihiro, and Jido et al.^{8,9,10} used such a model to calculate the even-odd parity nucleon mass difference as well as the decay properties of the odd-parity resonances as a function of their chiral transformation properties (“naive” or “mirror”).

In the QCD sum rule studies, it is well known that a typical combination of a scalar-isoscalar nucleon field Eq.(1) and of the pseudoscalar-isoscalar one Eq. (2) successfully describes the properties of the nucleon. In addition, this choice leads to one of the best known non-Abelian chiral transformation properties, *viz.* the “naive” non-Abelian one, while we find a somewhat complicated Abelian chiral transformation law (that can be reduced to a direct sum of a “triple naive” and a “mirror” Abelian chiral transformations). This is the starting assumption of this paper, and its consequences will be discussed extensively.

Let us consider the scalar-isoscalar ($\tilde{q}q$) and the pseudoscalar-isoscalar ($\tilde{q}\gamma^5 q$) even-parity nucleon fields

$$N_1^+ = \epsilon_{abc}(\tilde{q}_a q_b)q_c, \quad (6)$$

$$N_2^+ = \epsilon_{abc}(\tilde{q}_a \gamma^5 q_b) \gamma^5 q_c. \quad (7)$$

They transform according to the linear realization under the non-Abelian chiral transformations:

$$\delta_5^{\vec{a}} N_1^+ = i\gamma_5 \vec{\tau} \cdot \vec{a} N_1^+, \quad (8)$$

$$\delta_5^{\vec{a}} N_2^+ = i\gamma_5 \vec{\tau} \cdot \vec{a} N_2^+, \quad (9)$$

whereas under the Abelian chiral transformations the rule is also linear, but slightly more complicated as it mixes in the second nucleon field:

$$\delta_5 N_1^+ = ia\gamma_5(N_1^+ + 2N_2^+), \quad (10)$$

$$\delta_5 N_2^+ = ia\gamma_5(N_2^+ + 2N_1^+). \quad (11)$$

In other words they seem to form a two-dimensional representation of the Abelian chiral symmetry $U_L(1) \times U_R(1)$, or an $U_A(1)$ doublet. Of course, all irreducible representations of an Abelian Lie group are one-dimensional. Therefore the “chiral doublet” two-dimensional representation of $U_A(1)$ furnished by the fields $N_{1,2}$ and defined by Eqs. (14) and (11) must be a reducible one.

6 *Authors' Names*

The symmetric and antisymmetric linear combinations of two (identical parity) nucleon fields $N_{1,2}$ transform according to the irreducible representation:

$$N_n^+ = \frac{1}{\sqrt{2}}(N_1^+ + N_2^+), \quad (12)$$

$$N_m^+ = \frac{1}{\sqrt{2}}(N_1^+ - N_2^+). \quad (13)$$

Then their Abelian chiral transformation properties are

$$\delta_5 N_n^+ = 3ia\gamma_5 N_n^+, \quad (14)$$

$$\delta_5 N_m^+ = -ia\gamma_5 N_m^+. \quad (15)$$

Note the factor 3 in front of the r.h.s. of Eq. (14), i.e., it is the “triply-naive” Abelian axial baryon charge transformation law, as it should be for an object consisting of three quarks, and the negative sign in front of the r.h.s. of Eq. (15), as it should for an Abelian “mirror” nucleon. The non-Abelian transformations remain unchanged (“naive”)

$$\delta_5^a N_{m,n}^+ = i\gamma_5 \vec{\tau} \cdot \vec{a} N_{m,n}^+. \quad (16)$$

The triply-naive Abelian field does not have an $U_A(1)$ symmetric interaction that is linear in meson fields. The factor 3 on the right-hand side of the Abelian chiral transformation law Eq. (14) suggests, however, that the appropriate power of meson fields should also be three, and, indeed, there are two independent $U_A(1)$ invariants that are cubic in meson fields, see Appendix B. In other words, the $U_A(1)$ symmetry breaking is not intrinsic in the “triple-naive” Abelian nucleon structure.

Now, from the pair-wise nature of the nucleon fields under the $U_A(1)$ transformations, it is natural to consider the nucleon resonance states consisting of two parity doublets, i.e., of two even-parity and two odd-parity nucleons. If all the resonances belong to the naive non-Abelian representation, we can not avoid the decoupling of off-diagonal πNN interaction, as shown in Ref. ¹⁰. Hence we need to find two fields with “mirror” non-Abelian chiral properties.

We have shown that it is not possible to construct a “mirror” non-Abelian nucleon from three quarks without derivatives ⁶. Without such a “mirror” field, it is impossible to have a pure “naive-mirror mass term” that prevents the decoupling of the pion interaction term. Therefore, let us try and see if that can be done when one derivative is available.

2.2. *Odd-parity derivative nucleon fields*

We can construct nucleon fields with “contravariant” chiral transformations to those shown above by the replacement of γ_μ with a derivative $i\partial_\mu$ (or a covariant $iD_\mu = i\partial_\mu + eA_\mu$ in QCD), for example the following two derivative objects involving three quark fields

$$N_1'^- = \epsilon_{abc} i\partial_\mu (\tilde{q}_a q_b) \gamma^\mu \gamma^5 q_c, \quad (17)$$

$$N_2'^- = \epsilon_{abc} i\partial_\mu (\tilde{q}_a \gamma^5 q_b) \gamma^\mu q_c. \quad (18)$$

They are odd-parity, spin 1/2 and isospin 1/2 fields, i.e. they describe (some) nucleon resonances. A prime in the superscript implies that the fields contain a derivative, and we show below that therefore they have opposite, i.e., mirror, non-Abelian chiral transformation properties to those of the corresponding non-derivative fields.

Taking, once again, the symmetric and antisymmetric linear combinations of two (identical parity) nucleon fields $N_{1,2}'^-$ as the new canonical fields

$$N_m'^- = \frac{1}{\sqrt{2}}(N_1'^- + N_2'^-), \quad (19)$$

$$N_n'^- = \frac{1}{\sqrt{2}}(N_1'^- - N_2'^-), \quad (20)$$

their Abelian chiral transformation properties read

$$\delta_5 N_m'^- = -3ia\gamma_5 N_m'^-, \quad (21)$$

$$\delta_5 N_n'^- = ia\gamma_5 N_n'^-, \quad (22)$$

whereas the non-Abelian ones remain “mirror”

$$\delta_5^{\vec{a}} N_{m,n}'^- = -i\gamma_5 \vec{\tau} \cdot \vec{a} N_{m,n}'^-. \quad (23)$$

With these fields we are ready to construct “naive-mirror” interactions. We summarize the properties of the four fields in Table.1

Thus, we have constructed four independent nucleon fields: two fields with naive and two fields with mirror Abelian and non-Abelian chiral transformation properties. In the present paper, we identify these fields with the nucleon ground state $N(940)$ and its resonances $N(1440)$, $N(1535)$ and $N(1650)$.

Table 1. The axial charges of the nucleon fields. Two generic tentative assignments of physical states are shown as cases I and II. In case I both even-parity fields are non-Abelian naive, in case II the Roper is a mirror one.

	$U_A(1)$	$SU_A(2)$	(I)	(II)
N_m	-1	+1	$N(940)$	$N(940)$
N_n	+3	+1	$N(1440)$	$N(1535)$
N_n'	+1	-1	$N(1650)$	$N(1650)$
N_m'	-3	-1	$N(1535)$	$N(1440)$

3. Nucleon-meson chiral interactions

The previous studies^{8,9,10} developed a formalism based on one pair of naive and mirror opposite-parity nucleon fields. However, they did not make a reference to the $U_A(1)$ symmetry. Our strategy is first to construct the $SU_L(2) \times SU_R(2)$ chiral invariant interaction terms for $N_{m,n}^+$ and $N_{m,n}'^-$ fields. These terms are then classified according to their $U_A(1)$ symmetry. We shall see that besides the usual linear (in

meson fields) interactions there are also quadratic and cubic ones. Christos², on the other hand, has shown that there are two independent three-meson-one-nucleon interactions for each parity doublet that preserve both the Abelian and the non-Abelian chiral symmetry. That makes altogether six terms: four diagonal ones in the two doublets and two “inter-doublet” ones, see Appendix B. Furthermore, we can include quadratic terms that are allowed by the non-Abelian mirror properties of the baryons. We shall see that with so many $U_A(1)$ symmetry-conserving terms, we do not need any $U_A(1)$ symmetry breaking terms to describe this part of the nucleon mass spectrum, provided we use a complete set of interactions (see Sect. 3.2 below).

So far, we have considered identifications of $N_{m,n}^+$ with positive parity states, while $N_{m,n}'^-$ with negative parity states, as the operator containing a derivative may naturally describe orbital excitations. In principle, we can also consider the reversed case, however, where $N_{m,n}$ and $(N_{m,n}')^-$ are identified with negative (positive) parity states, respectively. In that case, the Lagrangians constructed in this sections for the former identification are transformed into those of the latter identification by multiplying all the nucleon fields with a γ_5 matrix, e.g., $N^- = \gamma_5 N^+$. We shall consider this possibility also in the next section.

In the following, we consider two parity doublets as follows; $\Psi = (N_m^+, N_n'^-)$ for the pair of the single Abelian charge (single Abelian doublet), and $\Phi = (N_n^+, N_m'^-)$ for that of the triple Abelian charge (triple Abelian doublet). We emphasize that the two nucleons of these pairs are in the “mirror” relations in both Abelian and non-Abelian chiral symmetries.

3.1. “Single-Abelian” doublet

First, we construct $U_A(1)$ symmetric Lagrangians from the nucleon fields with single Abelian charge N_m^+ and $N_n'^-$. Since N_m^+ is the naive for the non-Abelian chiral transformation, while $N_n'^-$ is the mirror, then the $SU_L(2) \times SU_R(2)$ invariant interaction terms up to first order of meson fields are as follows,

$$\bar{N}_m^+ A N_m^+, \bar{N}_m^+ B N_m^+, \bar{N}_n'^- A^\dagger N_n'^-, \bar{N}_n'^- B^\dagger N_n'^-,$$

where we have introduced the meson fields,

$$A = \sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi}, \quad (24)$$

$$B = \boldsymbol{\tau} \cdot \boldsymbol{\sigma} + i\gamma_5 \eta. \quad (25)$$

In addition, chiral invariant mass term is also possible, $\bar{N}_m^+ \gamma_5 N_n'^- + h.c.$ Then, the $U_A(1)$ symmetric Lagrangian is formed by a suitable combination of the interaction

terms as

$$\begin{aligned}
\mathcal{L}_{\text{int}}^{(1)} &= -g_1 \overline{N}_m^+ (A - B) N_m^+ - g_2 \overline{N}_n'^- (A^\dagger - B^\dagger) N_n'^- \\
&\quad - m_{12} [\overline{N}_m^+ \gamma_5 N_n'^- + \text{h.c.}], \\
&= -g_1 \overline{N}_m^+ [\sigma - \boldsymbol{\tau} \cdot \boldsymbol{\sigma} - i\gamma_5 (\eta - \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_m^+ \\
&\quad - g_2 \overline{N}_n'^- [\sigma - \boldsymbol{\tau} \cdot \boldsymbol{\sigma} + i\gamma_5 (\eta - \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_n'^- \\
&\quad - m_{12} [\overline{N}_m^+ \gamma_5 N_n'^- + \text{h.c.}]. \tag{26}
\end{aligned}$$

Here the relative minus sign between A and B has been chosen in order to preserve the $U_A(1)$ symmetry. Note that the g_1 and g_2 terms have different signs between σ and π fields that leads to differences in diagonalization and hence to pionic nucleon decays. This shows that it is possible to break the interlocking of mass and pion interaction matrices and thus allow for pionic decays of nucleon resonances¹⁰.

There are, moreover, three-meson interaction terms that preserve both Abelian and non-Abelian chiral symmetries^d

$$\begin{aligned}
\mathcal{L}_{\text{cubic int}}^{(1)} &= -g_3 f_\pi^{-2} \overline{N}_m^+ (AA^\dagger - BB^\dagger + AB^\dagger - BA^\dagger) (A + B) N_m^+ \\
&\quad - g_4 f_\pi^{-2} \overline{N}_n'^- (A^\dagger + B^\dagger) (AA^\dagger - BB^\dagger - AB^\dagger + BA^\dagger) N_n'^-, \\
&= -g_3 f_\pi^{-2} \overline{N}_m^+ [\sigma + \boldsymbol{\tau} \cdot \boldsymbol{\sigma} + i\gamma_5 (\eta + \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_m^+ (\sigma^2 - \boldsymbol{\sigma}^2 - \eta^2 + \boldsymbol{\pi}^2) \\
&\quad - 2g_3 f_\pi^{-2} \overline{N}_m^+ [i\gamma_5 (\sigma + \boldsymbol{\tau} \cdot \boldsymbol{\sigma}) - (\eta + \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_m^+ (\sigma\eta - \boldsymbol{\sigma} \cdot \boldsymbol{\pi}) \\
&\quad - g_4 f_\pi^{-2} \overline{N}_n'^- [\sigma + \boldsymbol{\tau} \cdot \boldsymbol{\sigma} - i\gamma_5 (\eta + \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_n'^- (\sigma^2 - \boldsymbol{\sigma}^2 - \eta^2 + \boldsymbol{\pi}^2) \\
&\quad + 2g_4 f_\pi^{-2} \overline{N}_n'^- [i\gamma_5 (\sigma + \boldsymbol{\tau} \cdot \boldsymbol{\sigma}) + (\eta + \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_n'^- (\sigma\eta - \boldsymbol{\sigma} \cdot \boldsymbol{\pi}). \tag{27}
\end{aligned}$$

After spontaneous symmetry breaking, $\sigma \rightarrow f_\pi + s$, however, this interaction leads to the same linearized meson-nucleon interaction term as the linear interaction Lagrangian Eq. (26) itself. Therefore for our present purposes, (*viz.* establishing the nucleon masses and two-body decay rates) the inclusion of this cubic term does not generate any differences and for this reason we shall henceforth drop this cubic interaction entirely.

As shown in Ref. 10,2 in some detail the linear effective Lagrangian Eq. (26) is capable of describing the $N(940)$ and the $N_n'^- = N^*(1535)$ or $N^*(1650)$ masses, while eliminating the single-pion decay $N_n'^- \rightarrow \pi N(940)$ and retaining the decay $N_n'^- \rightarrow \eta N(940)$.

3.2. “Triple-Abelian” doublet

Diagonal interactions for the nucleon fields with triple Abelian charge, N_n^+ and $N_m'^-$, is not as easily constructed, since there is no term such that it preserves the $U_A(1)$ symmetry and is linear in meson fields. As the desired interaction must be

^dWe use Christos’² construction here.

cubic in meson fields, Christos' ² construction has guided us in our quest.

$$\begin{aligned}
\mathcal{L}_{\text{cubic int}}^{(2)} = & -g_3 f_\pi^{-2} \overline{N}_n^+ (AA^\dagger - BB^\dagger - AB^\dagger + BA^\dagger) (A + B) N_n^+ \\
& -g_4 f_\pi^{-2} \overline{N}_m'^- (A^\dagger + B^\dagger) (AA^\dagger - BB^\dagger + AB^\dagger - BA^\dagger) N_m'^- \\
& -m_{34} [\overline{N}_n^+ \gamma_5 N_m'^- + \text{h.c.}], \\
= & -g_3 f_\pi^{-2} \overline{N}_n^+ [\sigma + \boldsymbol{\tau} \cdot \boldsymbol{\sigma} + i\gamma_5 (\eta + \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_n^+ (\sigma^2 - \boldsymbol{\sigma}^2 - \eta^2 + \boldsymbol{\pi}^2) \\
& -2g_3 f_\pi^{-2} \overline{N}_n^+ [i\gamma_5 (\sigma + \boldsymbol{\tau} \cdot \boldsymbol{\sigma}) - (\eta + \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_n^+ (\sigma\eta - \boldsymbol{\sigma} \cdot \boldsymbol{\pi}) \\
& -g_4 f_\pi^{-2} \overline{N}_m'^- [\sigma + \boldsymbol{\tau} \cdot \boldsymbol{\sigma} - i\gamma_5 (\eta + \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_m'^- (\sigma^2 - \boldsymbol{\sigma}^2 - \eta^2 + \boldsymbol{\pi}^2) \\
& +2g_4 f_\pi^{-2} \overline{N}_m'^- [i\gamma_5 (\sigma + \boldsymbol{\tau} \cdot \boldsymbol{\sigma}) + (\eta + \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_m'^- (\sigma\eta - \boldsymbol{\sigma} \cdot \boldsymbol{\pi}) \\
& -m_{34} [\overline{N}_n^+ \gamma_5 N_m'^- + \text{h.c.}]. \tag{28}
\end{aligned}$$

There are, of course, many $U_A(1)$ symmetry breaking terms. Their number, however, exceeds by far the number of available observables, so we cannot hope to fix them from experiments alone. More importantly, the $U_A(1)$ symmetry breaking terms are not necessary to explain the mass spectrum, nor the decays within each parity doublet. It is only the transitions/decays between members of the two doublets that need to be added by hand.

Note, however, that *if we had (incorrectly) insisted on interactions that are merely linear in meson fields, we would have been led to a different conclusion viz. that the masses of the “triple Abelian” nucleon parity doublet are degenerate in the good $U_A(1)$ symmetry limit.*

So, even though the mass splittings between the members of the parity doublets are of the same order of magnitude as the explicit $U_A(1)$ symmetry breaking scale, they may, in principle, be unrelated to this symmetry breaking. Only a detailed model calculation can distinguish between the symmetry-conserving and -breaking contributions.

3.3. Inter-doublet

We look at the inter-doublet interaction terms for the pairs of N_m^+ and $N_m'^-$, and N_n^+ and $N_n'^-$ ^e. There are two $U_A(1)$ symmetry invariants,

$$\begin{aligned}
\mathcal{L}_{\text{off-diag int}}^{(12)} = & -g_5 f_\pi^{-1} \left(\overline{N}_m^+ (AA^\dagger - BB^\dagger + AB^\dagger - BA^\dagger) \gamma_5 N_m'^- + \text{h.c.} \right) \\
& -g_6 f_\pi^{-1} \left(\overline{N}_n^+ (AA^\dagger - BB^\dagger - AB^\dagger + BA^\dagger) \gamma_5 N_n'^- + \text{h.c.} \right), \\
= & -g_5 f_\pi^{-1} \left(\overline{N}_m^+ [(\sigma^2 - \boldsymbol{\sigma}^2 - \eta^2 + \boldsymbol{\pi}^2) - 2i\gamma_5 (\sigma\eta - \boldsymbol{\sigma} \cdot \boldsymbol{\pi})] \gamma_5 N_m'^- + \text{h.c.} \right) \\
& -g_6 f_\pi^{-1} \left(\overline{N}_n^+ [(\sigma^2 - \boldsymbol{\sigma}^2 - \eta^2 + \boldsymbol{\pi}^2) + 2i\gamma_5 (\sigma\eta - \boldsymbol{\sigma} \cdot \boldsymbol{\pi})] \gamma_5 N_n'^- + \text{h.c.} \right). \tag{29}
\end{aligned}$$

^eThe naive-mirror pairings, e.g. N_m^+ and $N_n'^-$, or N_n^+ and $N_m'^-$ are trivial, for either parity combination.

There are no other (linear or cubic) meson interactions that maintain both Abelian and non-Abelian chiral symmetries and connect these two kinds of fields.

3.4. Intra-doublet

Next we consider the mixing between two non-Abelian-identical members of the two doublets: e.g. between N_m^+ and N_n^+ , or between $N_m'^-$ and $N_n'^-$. As the pion-nucleon interactions induced by these terms do not survive the mass diagonalization, they are of limited practical use. Because of this reason, we do not consider this type of interactions in the following phenomenological study. Here, however, we list them for the sake of completeness. There are two terms with one meson field, that are given as

$$\begin{aligned}\mathcal{L} = & g_7 [\bar{N}_n^+(A+B)N_m^+ + h.c.] \\ & + g_8 [\bar{N}_n'^-(A^\dagger+B^\dagger)N_m'^- + h.c.].\end{aligned}\quad (30)$$

There are also two $U_A(1)$ invariant terms with three meson fields, which are given as

$$\begin{aligned}\mathcal{L} = & g_9 [\bar{N}_n^+(AA^\dagger - BB^\dagger - AB^\dagger + BA^\dagger)(A-B)N_m + h.c.] \\ & + g_{10} [\bar{N}_n'^-(A^\dagger - B^\dagger)(AA^\dagger - BB^\dagger + AB^\dagger - BA^\dagger)N_m'^- + h.c.].\end{aligned}\quad (31)$$

3.5. $U_A(1)$ symmetry breaking terms

Finally and only for completeness' sake we consider the $U_A(1)$ symmetry breaking terms. Note that by changing the relative sign of the A and B in all of the previous terms we break the $U_A(1)$ symmetry while keeping the non-Abelian chiral symmetry intact. Thus, for each $U_A(1)$ symmetry conserving term there is (at least) one symmetry breaking term, and for cubic interactions more than one, as one can change the relative sign in several places. Thus, we see that for each $U_A(1)$ symmetry conserving term there is a symmetry breaking one, and that the two are indistinguishable with regard to their effects on the baryon masses and pion interactions. The “realistic” couplings contain both kinds of terms, of course.

3.6. Linearized nucleon-meson chiral interactions

Upon taking into account the spontaneous symmetry breaking $\sigma \rightarrow f_\pi + s$, we find the linearized forms of the linear, quadratic and cubic interactions

$$\begin{aligned}\mathcal{L}_{\text{int}}^{(1)} = & -g_1 \bar{N}_m^+ [f_\pi + s - \boldsymbol{\tau} \cdot \boldsymbol{\sigma} - i\gamma_5(\eta - \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_m^+ \\ & -g_2 \bar{N}_n'^- [f_\pi + s - \boldsymbol{\tau} \cdot \boldsymbol{\sigma} + i\gamma_5(\eta - \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_n'^- \\ & -m_{12} [\bar{N}_m^+ \gamma_5 N_n'^- + h.c.],\end{aligned}\quad (32)$$

and the linearized cubic term becomes

$$\begin{aligned}\mathcal{L}_{\text{lin cub int}}^{(1)} = & -g_3 \bar{N}_m^+ [f_\pi + 3s + \boldsymbol{\tau} \cdot \boldsymbol{\sigma} + i\gamma_5(3\eta + \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_m^+ \\ & -g_4 \bar{N}_n'^- [f_\pi + 3s + \boldsymbol{\tau} \cdot \boldsymbol{\sigma} - i\gamma_5(3\eta + \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_n'^-.\end{aligned}\quad (33)$$

Note that Eq. (32) and Eq. (33) have the same shape as far as the mass and $\vec{\pi}$ terms are concerned; the difference shows up only in the signs and sizes of the s and η interactions, which can be three times stronger, and of opposite sign. This means that we can put these two interactions together and call the effective interaction couplings g_1, g_2 . For this reason we have used g_3, g_4 to denote the second (“triple-Abelian”) doublet couplings and thus reduce the nomenclature clutter.

$$\begin{aligned} \mathcal{L}_{\text{lin cub int}}^{(2)} = & -g_3 \bar{N}_n^+ [f_\pi + 3s + \boldsymbol{\tau} \cdot \boldsymbol{\sigma} + i\gamma_5(3\eta + \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_n^+ \\ & -g_4 \bar{N}_m'^- [f_\pi + 3s + \boldsymbol{\tau} \cdot \boldsymbol{\sigma} - i\gamma_5(3\eta + \boldsymbol{\tau} \cdot \boldsymbol{\pi})] N_m'^- \\ & -m_{34} [\bar{N}_n^+ \gamma_5 N_m'^- + \text{h.c.}]. \end{aligned} \quad (34)$$

Finally, the linearized inter-doublet term is

$$\begin{aligned} \mathcal{L}_{\text{off-diag int}}^{(12)} = & -g_5 \left(\bar{N}_m'^- [f_\pi + 2s - 2i\gamma_5\eta] N_m^+ + \text{h.c.} \right) \\ & -g_6 \left(\bar{N}_n^+ [f_\pi + 2s + 2i\gamma_5\eta] N_n'^- + \text{h.c.} \right). \end{aligned} \quad (35)$$

As anticipated, the parity may be reversed, and N_m' may be assigned to be a positive parity field, for instance, the Roper resonance. Then the quadratic π interaction brings about the necessary $R \rightarrow \pi\pi N$ decay strength in the even-parity sector. The single-pion decay $R \rightarrow \pi N$ comes about due to the “diagonal” interactions (within each parity doublet). Still the odd-parity resonances $N^*(1535), N^*(1650)$ decays $N_m'^- \rightarrow \pi N$ and $N_m^- \rightarrow \pi R$ are forbidden in the good $U_A(1)$ symmetry limit, which can easily be corrected by including (many different) $U_A(1)$ symmetry breaking terms. This mechanism for Roper decay does not exist if we assume that the non-Abelian mirror field in the second doublet has odd parity.

3.7. Choosing the members of parity doublets

We have constructed the effective Lagrangian with the four baryons N_n, N_m, N_n' and N_m' . Now we proceed to the assignment of these fields with the physical nucleon resonances, which can be done by solving the diagonalization of the 4×4 mass matrix. Of course, a complete exact diagonalization ought to lead to the same (unique) solution, no matter what starting point one adopts. That statement, however, holds only in the idealized world in which all decays are kinematically allowed and have been measured. Needless to say, we do not live in such a world, so we must employ various tactics.

In the present paper, we rather employ a simple method with the use of some insight from the lattice QCD and/or QCD sum rules, which tell us that some observed baryons are dominated by particular types of the baryon fields.

We begin with the classification of the four baryons into two doublets $\Psi = (N_m, N_n')$ and $\Phi = (N_n, N_m')$ or their admixtures, with actual resonances *viz.* $N(940), R(1440) N^*(1535)$ and $N^*(1650)$. Though having a larger number of variations, we consider two essentially different scenarios.

As stated in the Introduction, a substantial body of QCD sum rule evidence is pointing towards $N(940)$ as being the “Ioffe current” operator N_m^+ . Together with the lowest negative-parity nucleon $N(1535)$ as the partner in the parity doublet, we have $\Psi = (N_m^+, N_n'^-) = (N(940), N(1535))$ and $\Phi = (N_n^+, N_m'^-) = (N(1440), N(1650))$. This is **Scenario I**.

In another choice, we attempt to identify the negative parity state $N(1535)$ with the non-derivative field N_n^- , as the QCD sum rule implies substantial strength of the coupling between the ground state. Hence we have $\Psi = (N_m^+, N_n'^-) = (N(940), N(1650))$ and $\Phi = (N_n^-, N_m'^+) = (N(1535), N(1440))$. This is **Scenario II**.

This way of assigning the fields to states, rather than blind solving the full 4×4 mass matrices, may give us some insight into the physical nature of the potential solution(s). Now, we shall attempt to estimate the free parameters in each case, so as to determine viability of either scenario.

4. Results

4.1. Masses

Chiral symmetry is spontaneously broken through the “condensation” of the sigma field $\sigma \rightarrow \sigma_0 = \langle \sigma \rangle_0 = f_\pi$, which leads to the dynamical generation of baryon masses, as can be seen from the linearized chiral invariant interaction Lagrangians Eqs. (32)-(35). Each of the two parity doublets separately obeys the mass formulas given in Ref. ¹⁰ for the “mirror” case, provided there is no interaction between the two doublets. We shall assume this at first as the zeroth approximation, just to get a qualitative feel for the results we may expect. There is additional mixing in the four-nucleon mass/interaction matrix, however, due to the “inter-doublet” interaction Eq.(29).

• Scenario I

The nucleon mass matrix is already in a simple block-diagonal form when the nucleon fields form the following 1×4 row/column “vector”: $(\Psi, \Phi) = (N_m^+, N_n'^-, N_n^+, N_m'^-) \rightarrow (N_m^+, \gamma_5 N_n'^-, N_n^+, \gamma_5 N_m'^-)$

$$\lim_{U_A(1)\text{symm.}} M = \begin{pmatrix} g_1 f_\pi & m_{12} \gamma_5 & 0 & g_5 f_\pi \gamma_5 \\ m_{12} \gamma_5 & g_2 f_\pi & g_6 f_\pi \gamma_5 & 0 \\ 0 & g_6 f_\pi \gamma_5 & g_3 f_\pi & m_{34} \gamma_5 \\ g_5 f_\pi \gamma_5 & 0 & m_{34} \gamma_5 & g_4 f_\pi \end{pmatrix}. \quad (36)$$

Note that only the parity-changing interaction $g_{5,6} \neq 0$ mixes these two new equal parity doublets. Manifestly we may divide our analysis into two parts: one with, and another without parity-flipping coupling.

First note that upon redefinition of odd-parity fields with a γ_5 , as in $N_i' = \gamma_5 N_i^-$, where $i = 1, 2$, the masses of the redefined fields pick up a minus sign. This means that two of the mass eigenvalues will be negative. Proper

mass sign is restored at the end of the calculation when one reverts back to odd parity fields, this time to diagonalized ones, however.

The off-diagonal parity-non-changing coupling terms ($g_{5,6}$) in the mass matrix do not appear to improve our ability to fit this spectrum: rather they only seem to complicate the fitting procedure. We shall set them equal to zero at first and use them only later as they become necessary to fit the decay properties. Without inter-doublet interactions ($g_{5,6} = 0$) one can immediately read off the eigenvalues:

$$M_{\pm}^{(1)} = \frac{1}{2} \left[\sqrt{(g_1 + g_2)^2 f_{\pi}^2 + 4m_{12}^2} \pm (g_1 - g_2) f_{\pi} \right], \quad (37)$$

$$M_{\pm}^{(2)} = \frac{1}{2} \left[\sqrt{(g_3 + g_4)^2 f_{\pi}^2 + 4m_{34}^2} \pm (g_3 - g_4) f_{\pi} \right], \quad (38)$$

where the former two, Eq. (37), correspond to the first (nucleon) parity doublet and the latter two Eq. (38), correspond to the second (Roper) parity doublet. Following Ref. ¹⁰, we can determine the coupling and mass parameters, as well as the mixing angles θ_{ij} , determined by

$$\tan 2\theta_{ij} = \frac{2m_{ij}}{(g_i + g_j)f_{\pi}}. \quad (39)$$

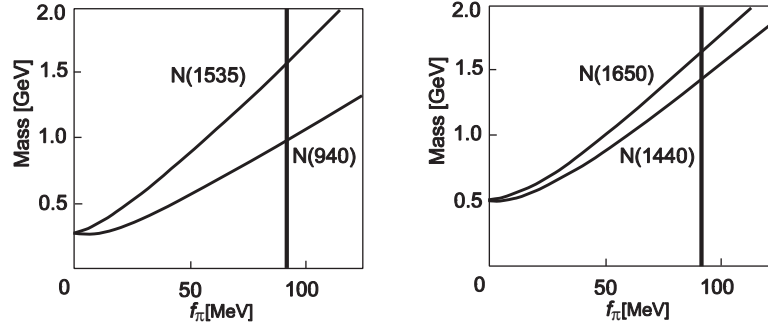
We show the results in Table 2 and Fig. 1.

Table 2. Coupling constants obtained from the nucleon masses with doublets ($N(940)$, $N^*(1535)$), ($R(1440)$, $N^*(1650)$) and the decay widths $N^*(1535) \rightarrow \pi N(940)$ and $N^*(1650) \rightarrow \pi R(1440)$. (Scenario I)

constant	value
g_1	10.4
g_2	16.8
m_{12}	270 MeV
θ_{12}	6.3°
g_3	14.6
g_4	16.8
m_{34}	503 MeV
θ_{34}	9.5°

• Scenario II

Once again, the nucleon mass matrix is in a simple block-diagonal form when the nucleon fields form the following 1×4 row/column "vector":

Fig. 1. The nucleon masses as functions of $\langle\sigma\rangle_0 = f_\pi$.Table 3. Coupling constants obtained from the nucleon masses with doublets ($N(940)$, $N^*(1650)$), ($R(1440)$, $N^*(1535)$) and the decay widths $N^*(1650) \rightarrow \pi N(940)$ and $N^*(1535) \rightarrow \pi R(1440)$. (Scenario II)

constant	value
g_1	10.5
g_2	18.1
m_{12}	295 MeV
θ_{12}	6.8°
g_3	—
g_4	—
m_{34}	—
θ_{34}	—

$$(\Psi, \Phi) = (N_m^+, N_n^-, N_m'^+, N_n^-) \rightarrow (N_m^+, \gamma_5 N_n'^-, N_m'^+, \gamma_5 N_n^-).$$

$$\lim_{U_A(1)\text{symm.}} M = \begin{pmatrix} g_1 f_\pi & m_{12} \gamma_5 & g_5 f_\pi & 0 \\ m_{12} \gamma_5 & g_2 f_\pi & 0 & g_6 f_\pi \\ g_5 f_\pi & 0 & g_3 f_\pi & m_{34} \gamma_5 \\ 0 & g_6 f_\pi & m_{34} \gamma_5 & g_4 f_\pi \end{pmatrix}. \quad (40)$$

Note that only the inter-parity-doublet interactions $g_{5,6} \neq 0$ mix these two parity-doublets. We may repeat our analysis as in the first case, but the data is insufficient to determine all the couplings as in the Scenario I. We show the results in Table 3. Note that here we do not attempt to evaluate the coupling constants g_3 and g_4 , because the decay of $N(1535)$ to $R(1440)\pi$ is kinematically forbidden when using the central values of the resonance masses.

Next, we remember that the Roper mixes with the ground state as well, if it is a non-Abelian mirror field, as in scenario II. In other words, one must take into

Table 4. Coupling constants obtained from the nucleon masses of the equal-parity doublet ($N(940)$, $R(1440)$).

constant	value
g_1	15.15
g_3	10.45
g_5	-1.3
θ_{13}	14.5°

account the equal-parity naive-mirror mixing due to $g_5 \neq 0$. We show the results in Table 4. Here

$$\tan 2\theta_{13} = \frac{-2g_5}{(g_1 - g_3)}. \quad (41)$$

The predicted coupling strength $g_{\pi NR} = \frac{1}{2}(g_3 - g_1) \sin 2\theta_{13}$ is significantly smaller than the one obtained from the decay width $R(1440) \rightarrow \pi N(940)$. The order of magnitude of g_1 is the same in both fits, so we may conclude that a simultaneous diagonalization of the complete mass matrix may lead to complete agreement.

Manifestly, the good $U_A(1)$ symmetry limit is sufficient to reproduce the nucleon spectrum in either scenario. Thence our **main conclusion**: *mass degeneracy of opposite-parity nucleon resonances is not a consequence of the explicit $U_A(1)$ symmetry (non) breaking*. This conclusion was also reached by Christos, albeit for just one parity doublet and without mirror fields. In general one has four (quadratic) equations with at least eight unknowns (six coupling constants and two bare “masses”). Clearly one needs other input, e.g. the decay widths, to fix all six parameters. There are too few measured/able decay widths to fix this ambiguity, however. A complete solution of this problem is beyond the realm of this paper, anyway.

4.2. The axial couplings

The mixing of naive and mirror nucleons leads to a change of the axial coupling constants, both isovector $g_A^{(1)}$ and isoscalar $g_A^{(0)}$. The simple mixing due to the “mirror mass” term m_{12} can only reduce the absolute value of both axial coupling constants from unity, in both scenarios. The mixing angles θ_{ij} are shown in Tables 2 and 4, which leads to

$$g_A^{(1)} = \cos^2 \theta_{12} - \sin^2 \theta_{12} = 0.976, \quad (42)$$

$$g_A^{(0)} = \sin^2 \theta_{12} - \cos^2 \theta_{12} = -0.976, \quad (43)$$

in Scenario I, and

$$g_A^{(1)} = \cos^2 \theta_{12} - \sin^2 \theta_{12} = 0.972, \quad (44)$$

$$g_A^{(0)} = \sin^2 \theta_{12} - \cos^2 \theta_{12} = -0.972, \quad (45)$$

Table 5. Axial coupling constants obtained in different scenarios.

constant	I	II	II.A
$g_A^{(1)}$	0.976	0.972	0.875
$g_A^{(0)}$	- 0.976	- 0.972	- 1.125

in Scenario II. Manifestly, neither of these two values of the isoscalar axial coupling constant is anywhere close to the measured one, $g_A^{(0)} = 0.28 \pm 0.16$.

On the other hand, the nucleon-Roper mixing angle θ_{13} , due to the off-diagonal coupling g_5 in Scenario II changes the value of the isoscalar $g_A^{(0)}$ to somewhere between its “Ioffe” value of -1 and -3 :

$$g_A^{(1)} = \cos^2 \theta_{13} - \sin^2 \theta_{13} = \cos 2\theta_{13} = 0.875, \quad (46)$$

$$g_A^{(0)} = -3 \sin^2 \theta_{13} - \cos^2 \theta_{13} = -2 + \cos 2\theta_{13} = -1.125, \quad (47)$$

Unfortunately we have already seen that the analogous mixing angle θ_{14} in Scenario I can not be determined from the present analysis, but whatever its value, it would only improve the value of

$$g_A^{(0)} = 3 \sin^2 \theta_{14} - \cos^2 \theta_{14} = 1 - 2 \cos 2\theta_{14} \geq -1. \quad (48)$$

We summarize the situation in Table 5. We are forced to conclude that none of these scenarios lead to a viable picture of the nucleon ground state (though perhaps some may be viable for the resonances). Of course, we have not included the mixing with the $(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)$ chiral multiplet, as yet, which was assumed by Weinberg¹ to be vital for the isovector axial coupling, and may yet solve the isoscalar axial coupling problem, as well¹¹.

5. Summary and Discussion

We have analyzed the role of chiral symmetry in general and of the $U_A(1)$ symmetry in particular in the nucleon-Roper-two-odd-parity-nucleon-resonances system, under the assumption that the above four nucleon states form a particular set of chiral multiplets, as implied by the three-quark construction of the baryon interpolating fields. The four nucleon fields naturally split into two “parity doublets” due to their $U_A(1)$ symmetry transformation properties. We classify the meson-nucleon interactions according to their $U_A(1)$ symmetry transformation properties. It is crucial to keep all $U_A(1)$ symmetry conserving interaction terms, even the “cubic” ones, which are sometimes redundant for the purpose of mass determination. Yet, note, that *if we had only (incorrectly) insisted on interactions that are linear in meson fields, we would have been led to the different conclusion that the masses of the “triple Abelian” nucleon parity doublet are degenerate in the good $U_A(1)$ symmetry limit.*

The nucleon mass spectrum and the one-pion decay properties have been used to fix some of the free coupling constants in the (σ, π) sector, see Tables 2- 4. Only

two $N^* \rightarrow \eta + N$ decays are kinematically allowed, so they are not sufficient to determine (all of) the remaining $(\bar{\sigma}, \eta)$ couplings.

The insight that the nucleon and the Roper fields may form two different representations of the $U_A(1)$ symmetry, and that their mass differences can be explained only by the spontaneous breaking of $SU_L(2) \times SU_R(2)$ and $U_A(1)$ symmetries, while explicitly preserving the $U_A(1)$ symmetry, is the main result of the present paper. A corollary of this result is that the parity-doublet mass splittings are not determined by the $U_A(1)$ symmetry breaking, as was conjectured in the literature¹⁷. Moreover, the nucleon-Roper mass difference in some calculations, such as the one of Ref.¹⁴ in the NJL model, are not a consequence of the broken $U_A(1)$ symmetry in that model, either.

$U_A(1)$ symmetry in nucleon spectra has been discussed before, most notably by Christos², who used only one parity doublet ($N(940)$ and $N^*(1535)$), however, and drew conclusions that are consistent with, but only a small subset of ours. He argued that the parity doublet mass difference is proportional to a particular ηNN^* coupling constant, which is in close agreement with our results. He did not try to connect other mass differences, such as the Roper-nucleon one, to this mechanism, as he did not use alternative (“mirror”) sets of fields, which is a novel contribution of our paper (for a comparison of our formalism with Christos’, see Appendix A).

Jido, Oka and one of us (A.H.)⁷ studied QCD sum rules for the odd-parity nucleon resonance $N^*(1535)$ as a function of the field and its $U_A(1)$ transformation property. We found that this transformation property is the crucial ingredient determining the ηNN^* coupling constant. This was perhaps the first explicit demonstration of the $U_A(1)$ symmetry’s role in the odd-parity nucleon spectra; Jaffe *et al.* have argued for the same goal, but along different, more general lines¹⁷: It is well known that spontaneously broken symmetry, like the $SU_L(2) \times SU_R(2)$ one, cannot lead to mass splitting predictions without additional assumptions^{15,17}. $U_A(1)$ symmetry is different in this regard as it is explicitly broken, and badly at that. Different operators break this symmetry in different ways and this difference might show up in the mass spectra. An explicitly broken linear Abelian chiral symmetry such as the $U_A(1)$ one, can predict mass relations in certain special situations, however, as was shown in the case of scalar mesons in Ref.¹⁶. In the nucleon-meson problem, however, there is a sufficiently large latitude to allow a fit of the four nucleon masses and of the most important nucleon decays without having to invoke the explicit $U_A(1)$ symmetry breaking.

Acknowledgments

We wish to thank Dr. N. Ishii and Prof. W. Bentz, for valuable conversations about the Fierz transformation of nucleon fields and (in)dependence of nucleon fields. One of us (V.D.) thanks Prof. H. Toki for hospitality at RCNP, where this work was started, on two occasions and Prof. S. Fajfer for warm hospitality at the Institute Jožef Stefan in Ljubljana where this work was finished.

Appendix A. Comparison with Christos, Ref. [**]

Let us consider the two baryon operators,

$$B^1 = (q_a^T C \gamma^5 i \tau_2 q_b) q_c \epsilon_{abc}, \quad (\text{A.1})$$

$$B^2 = (q_a^T C \gamma^5 i \tau_2 q_b) \gamma_5 q_c \epsilon_{abc}. \quad (\text{A.2})$$

Here indices a , b and c label the color of the three quarks, whereas the Pauli matrices τ_i operate in the flavor space. We shall omit the color index from now on. The choice of the second operator is different from the original choice by Christos², who employed

$$B^3 = (q^T C i \tau_2 q) q. \quad (\text{A.3})$$

(In the original paper, B^3 was labelled B^2 and *vice versa*.) Spin and parity of B^2 and B^3 are the same.

Christos² considered the decomposition of $B^{1,3}$ fields in the $L - R$ representation. The operators B^1 and B^3 are reduced to

$$\begin{aligned} B_L^1 &= [-q_L^T C i \tau_2 q_L + q_R^T C i \tau_2 q_R] q_L \\ B_R^1 &= [-q_L^T C i \tau_2 q_L + q_R^T C i \tau_2 q_R] q_R \\ B_L^3 &= [q_L^T C i \tau_2 q_L + q_R^T C i \tau_2 q_R] q_L \\ B_R^3 &= [q_L^T C i \tau_2 q_L + q_R^T C i \tau_2 q_R] q_R \end{aligned}$$

We see that in the $L - R$ representation, there are four independent operators,

$$[q_L^T C i \tau_2 q_L] q_L, [q_R^T C i \tau_2 q_R] q_L, [q_L^T C i \tau_2 q_L] q_R, [q_R^T C i \tau_2 q_R] q_R. \quad (\text{A.4})$$

It is important that the four operators $B_{L,R}^{1,3}$ are independent (different) as expressed by the four combinations of the four operators in $L - R$ representation. In other words, we may rewrite the four independent operators in the $L - R$ representation by way of $B_{L,R}^{1,3}$. For example,

$$[q_L^T C i \tau_2 q_L] q_L = (B_L^3 - B_L^1)/2. \quad (\text{A.5})$$

All four operators of Eq. (A.4) can be expressed in terms of $B_{L,R}^{1,3}$, none of which are identical. Christos constructed the chiral Lagrangian by using the operators B_1 and B_3 .

Next we consider the same algebra with operators Eq. (A.2),

$$B_L^1 = [-q_L^T C i \tau_2 q_L + q_R^T C i \tau_2 q_R] q_L \quad (\text{A.6})$$

$$B_R^1 = [-q_L^T C i \tau_2 q_L + q_R^T C i \tau_2 q_R] q_R \quad (\text{A.7})$$

$$B_L^2 = -[-q_L^T C i \tau_2 q_L + q_R^T C i \tau_2 q_R] q_L \quad (\text{A.8})$$

$$B_R^2 = [-q_L^T C i \tau_2 q_L + q_R^T C i \tau_2 q_R] q_R \quad (\text{A.9})$$

In this case we find $B_R^1 = B_R^2$ and $B_L^1 = -B_L^2$. Thus, we can not represent four operators in $L - R$ representation with $B_{L,R}^{1,2}$ independently. Hence, we can not apply Christos' method to operators B_1 and B_2 .

This tells us that only four helicity components are independent: a pair of L- and R-handed baryon operators each. As each massive Dirac field requires two chiral components we conclude that there are (only) two independent nucleon operators, of either parity. The parity of the field can be arbitrarily chosen and the opposite parity field (the parity partner) is necessarily degenerate.

Appendix B. Higher order meson terms in the interaction Lagrangian

We consider higher order terms with containing two- and three-meson fields. The linear meson terms was defined in Eq.(25) as,

$$\begin{aligned} A &= [\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi}], \\ B &= [\boldsymbol{\tau} \cdot \boldsymbol{\sigma} + i\gamma_5 \eta]. \end{aligned}$$

Under $SU(2)_A$ transformation, these terms transform into

$$A \rightarrow U^\dagger A U^\dagger, \quad B \rightarrow U^\dagger B U^\dagger. \quad (\text{B.1})$$

Both of them are covariant under $SU(2)_A$. In contrast they are not covariant under $U(1)_A$, however we can construct covariant terms by choosing the linear combinations of them as,

$$\delta_5(A + B) \rightarrow -2i\gamma_5 a(A + B), \quad (\text{B.2})$$

$$\delta_5(A - B) \rightarrow +2i\gamma_5 a(A - B). \quad (\text{B.3})$$

Higher power terms are constructed by these linear terms. The term with two-meson fields are given as

$$M_2 = AA^\dagger - BB^\dagger + AB^\dagger - BA^\dagger, \quad (\text{B.4})$$

$$M_3 = AA^\dagger - BB^\dagger - AB^\dagger + BA^\dagger, \quad (\text{B.5})$$

These terms are reexpressed by using the determinant term with $M = \bar{q}_i(1 - \gamma_5)q_j$, as

$$AA^\dagger - BB^\dagger = \frac{1}{2}(\det M + \det M^\dagger), \quad (\text{B.6})$$

$$A^\dagger B - B^\dagger A = \frac{1}{2}\gamma_5(\det M - \det M^\dagger), \quad (\text{B.7})$$

or explicitly,

$$AA^\dagger - BB^\dagger = \sigma^2 + \pi^2 - \eta^2 - \vec{\sigma}^2, \quad (\text{B.8})$$

$$A^\dagger B - B^\dagger A = 2i\gamma_5(\sigma\eta - \vec{\sigma} \cdot \vec{\pi}). \quad (\text{B.9})$$

The transformation under $SU(2)_V \times SU(2)_A$ are

$$M_i \rightarrow U^\dagger M_i U, \quad U = e^{i\boldsymbol{\tau} \cdot \boldsymbol{a} \gamma_5}, \quad (\text{B.10})$$

where $i = 2, 3$. Note that the difference between the transformations M_i and A, B , by which the non-Abelian naive-mirror mixing terms can be constructed. Under $U_A(1)$ transformation, these terms transform into

$$\delta_5 M_2 = 4i\gamma_5 a M_2, \quad (\text{B.11})$$

$$\delta_5 M_3 = -4i\gamma_5 a M_3. \quad (\text{B.12})$$

Finally, we construct the cubic meson terms by the combinations of the linear and square terms. The $SU(2)_L \times SU(2)_R$ symmetry allows the following terms to be covariant,

$$\begin{aligned} M_i(A \pm B) &\rightarrow U^\dagger M_i(A \pm B)U^\dagger, \\ (A^\dagger \pm B^\dagger)M_i &\rightarrow U(A^\dagger \pm B^\dagger)M_i U. \end{aligned} \quad (\text{B.13})$$

Of course their hermite conjugates can also be used. However there is a relation $M_2^\dagger = M_3$ and $M_3^\dagger = M_2$. Then, the hermite conjugate of the second of Eqs.(B.13) gives just the former one. Hence there are eight cubic meson terms. Finally, we summarize the $U_A(1)$ transformation properties of the cubic terms.

$$\delta_5 M_2(A + B) \rightarrow 2i\gamma_5 a M_2(A + B), \quad (\text{B.14})$$

$$\delta_5 M_3(A + B) \rightarrow -6i\gamma_5 a M_3(A + B), \quad (\text{B.15})$$

$$\delta_5 M_2(A - B) \rightarrow 6i\gamma_5 a M_2(A - B), \quad (\text{B.16})$$

$$\delta_5 M_3(A - B) \rightarrow -2i\gamma_5 a M_3(A - B), \quad (\text{B.17})$$

$$\delta_5(A^\dagger + B^\dagger)M_2 \rightarrow 6i\gamma_5 a(A^\dagger + B^\dagger)M_2, \quad (\text{B.18})$$

$$\delta_5(A^\dagger + B^\dagger)M_3 \rightarrow -2i\gamma_5 a(A^\dagger + B^\dagger)M_3, \quad (\text{B.19})$$

$$\delta_5(A^\dagger - B^\dagger)M_2 \rightarrow 2i\gamma_5 a(A^\dagger - B^\dagger)M_2, \quad (\text{B.20})$$

$$\delta_5(A^\dagger - B^\dagger)M_3 \rightarrow -6i\gamma_5 a(A^\dagger - B^\dagger)M_3. \quad (\text{B.21})$$

References

1. S. Weinberg, Phys. Rev. **177** (1969) 2604.
2. G.A. Christos, Z. Phys. **C 21**, 83 (1983);
3. G.A. Christos, Phys. Rev. **D 35**, 330 (1987).
4. H. q. Zheng, "Singlet axial coupling, proton structure and the parity doublet model," CERN-TH-6327-91 preprint (unpublished); H. q. Zheng, "Chiral Theory With 1/2-Octet Baryons," CERN-TH-6522-92 preprint (unpublished).
5. Hua-Xing Chen, V. Dmitrašinović, A. Hosaka, K. Nagata and S. L. Zhu, Phys. Rev. **D 78**, 054021(17) (2008).
6. K. Nagata, A. Hosaka and V. Dmitrasinovic, Eur. Phys. J. C **57**, 557 (2008); see also V. Dmitrašinović, K. Nagata and A. Hosaka, Mod. Phys. Lett. **A 23**, No.27/30, 2381-2384 (2008).
7. D. Jido, M. Oka and A. Hosaka, Phys. Rev. Lett. **80**, 448 (1998).
8. B. W. Lee, *Chiral Dynamics*, (Gordon and Breach, New York, 1972).
9. C. DeTar and T. Kunihiro, Phys. Rev. D **39**, 2805 (1989).
10. D. Jido, M. Oka and A. Hosaka, Prog. Theor. Phys. **106**, 873 (2001).
11. V. Dmitrašinović, A. Hosaka and K. Nagata, "Baryon chiral multiplet mixing: $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ and $(1, \frac{1}{2}) \oplus (1, \frac{1}{2})$ ", in preparation (2008).
12. B.L. Ioffe, Nucl. Phys. **B 188**, 317 (1981); *ibid.* **B 191**, 591 (1981) (E); Z. Phys. **C 18**, 67 (1983).
13. D. Jido, N. Kodama and M. Oka, Phys. Rev. D **54**, 4532 (1996) [arXiv:hep-ph/9604280].
14. Keitaro Nagata, Atsushi Hosaka, hep-ph/0506193; Keitaro Nagata, Atsushi Hosaka, Laith J. Abu-Raddad, Phys.Rev.C72:035208 (2005), (E)-*ibid.*C73:049903 (2006).
15. S. Weinberg, Phys. Rev. **166**, 1568 (1966).
16. V. Dmitrašinović, Phys. Rev. **C 53**, 1383 (1996), Nucl. Phys. **A 686**, 377 (2001); E. Klempt, B. C. Metsch, C. R. Münz, and H. R. Petry, Phys. Lett. **B 361**, 160 (1995); M. Hirata and N. Tsutsui, Phys. Rev. **D 56**, 5696 (1997); M. Takizawa, and T. Umekawa, Prog. Theor. Phys. **109**, 969 (2003) and T. Umekawa, K. Naito, M. Oka, and M. Takizawa, Phys.Rev. **C 70**:055205 (2004).
17. R.L. Jaffe, D. Pirjol, A. Scardicchio, hep-ph/051108, and R.L. Jaffe, D. Pirjol, A. Scardicchio, hep-ph/0602010.